# Convolutional Neural Networks I 

CS 4391 Introduction to Computer Vision
Professor Yapeng Tian
Department of Computer Science

## Visual Perception vs. Computational Perception



High-level information

- Depth
- Motion
- Object classes
- Object poses
- Etc.


## Mathematic Models

Try to model the human brain with computational models, e.g., neural networks


## Mathematic Models

What is the form of the function $f(x)$ ?

- No idea!
- Concatenate simple functions (neurons)



## Neural Network: Concatenation of functions

Linear score function: $f=W$
2-layer Neural Network

$$
f=f_{2}\left(f_{1}(x)\right)=W_{2} \max _{\text {Non-linearity }}\left(0, W_{1} x\right)
$$



## Activation Functions

2-layer Neural Network

$$
f=f_{2}\left(f_{1}(x)\right)=W_{2} \max \left(0, W_{1} x\right)
$$

rectified linear unit (ReLU)
$\max (0, x)$


Introduce non-linearity to the network

## Activation Functions

## Sigmoid <br> $$
\sigma(x)=1 /\left(1+e^{-x}\right)
$$


$\tanh \tanh (\mathrm{x})$

$$
\frac{e^{2 x}-1}{e^{2 x}+1}
$$

Hyperbolic tangent

ReLU $\max (0, x)$

Leaky ReLU max(0.1x, x)


Maxout $\max \left(w_{1}^{T} x+b_{1}, w_{2}^{T} x+b_{2}\right)$

ELU Exponentia $f(x)= \begin{cases}x & \text { if } x>0 \\ \text { Linear Unit } & \text { (exp }(x)-1) \\ \text { if } x \leq 0\end{cases}$


## Fully Connected Layer



## Fully Connected Layer

What is the drawback of only using fully connected layers?

$$
y=W x
$$

Consider an image with $640 \times 480$

- $x$ is with dimension 307,200
- The weight matrix of the fully connect layer is too large


## Convolutional Layers

## Consist of convolutional filters

Share weights among different image locations

$$
g(x, y)=\frac{1}{2 \pi \sigma^{2}} e^{-\frac{x^{2}+y^{2}}{2 \sigma^{2}}}
$$

Gaussian
Filter


Learn the weights!

## Convolutional Neural Networks



> [LeNet-5, LeCun 1980]

## Convolutional Neural Networks



## Convolutional Layer

$32 \times 32 \times 3$ image


## Convolutional Layer

$32 \times 32 \times 3$ image


Convolve the filter with the image i.e. "slide over the image spatially, computing dot products"

## Convolutional Layer



## Convolutional Layer

activation map


## A closer look at spatial dimensions:

7


## $7 \times 7$ input (spatially) assume $3 \times 3$ filter, with stride 1

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## $7 \times 7$ input (spatially) assume $3 \times 3$ filter

A closer look at spatial dimensions:

7

$7 \times 7$ input (spatially)
assume $3 \times 3$ filter
=> $5 \times 5$ output

A closer look at spatial dimensions:

7

$7 \times 7$ input (spatially)
assume $3 \times 3$ filter
applied with stride 2

A closer look at spatial dimensions:

7

$7 \times 7$ input (spatially)
assume $3 \times 3$ filter
applied with stride 2

A closer look at spatial dimensions:

$7 \times 7$ input (spatially) assume $3 \times 3$ filter applied with stride 2
=> $3 \times 3$ output!

Output size:
( N - F ) / stride + 1

A closer look at spatial dimensions:

7

$7 \times 7$ input (spatially)
assume $3 x 3$ filter
applied with stride $3 ?$

A closer look at spatial dimensions:

7

$7 \times 7$ input (spatially)
assume $3 \times 3$ filter
applied with stride $3 ?$

A closer look at spatial dimensions:

7

$7 \times 7$ input (spatially) assume $3 \times 3$ filter applied with stride 3 ?

## doesn't fit!

cannot apply $3 \times 3$ filter on $7 \times 7$ input with stride 3 .


## Output size: <br> ( N - F) / stride + 1

$$
\begin{aligned}
& \text { e.g. } N=7, F=3 \text { : } \\
& \text { stride } 1=>(7-3) / 1+1=5 \\
& \text { stride } 2=>(7-3) / 2+1=3 \\
& \text { stride } 3=>(7-3) / 3+1=2.33
\end{aligned}
$$

In practice: Common to zero pad the border

e.g. input $7 \times 7$
$3 \times 3$ filter, applied with stride 1
pad with 1 pixel border => what is the output?
(recall:)
( $\mathrm{N}-\mathrm{F}$ ) / stride +1

In practice: Common to zero pad the border

e.g. input $7 \times 7$
$3 \times 3$ filter, applied with stride 1
pad with 1 pixel border => what is the output?

7x7 output!

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## 7x7 output!

in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)
e.g. $F=3=>$ zero pad with 1
$F=5=>$ zero pad with 2
F = 7 => zero pad with 3

A closer look at spatial dimensions:
activation map


## Convolutional Layer consider a second, green filter



For example, if we had $65 \times 5$ filters, we'll get 6 separate activation maps:
activation maps


We stack these up to get a "new image" of size $28 x 28 x 6$ !

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions


Preview: ConvNet is a sequence of Convolutional Layers, interspersed with activation functions


## Convolutional Neural Networks



## Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



## MAX POOLING

Single depth slice


## Training: back-propotate errors




## Case Study: LeNet-5

[LeCun et al., 1998]


Conv filters were $5 \times 5$, applied at stride 1
Subsampling (Pooling) layers were $2 \times 2$ applied at stride 2
i.e. architecture is [CONV-POOL-CONV-POOL-CONV-FC]

## Case Study: AlexNet

[Krizhevsky et al. 2012]


Input: 227x227x3 images
First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Q: what is the output volume size? Hint: (227-11)/4+1 = 55

## Case Study: AlexNet

[Krizhevsky et al. 2012]


Input: 227x227x3 images
First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume [55x55x96]
Q: What is the total number of parameters in this layer?

## Case Study: AlexNet

[Krizhevsky et al. 2012]


Input: 227x227x3 images
First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume [55x55x96]
Parameters: $\left(11^{*} 11 * 3\right)^{*} 96=35 \mathrm{~K}$

## Case Study: AlexNet

[Krizhevsky et al. 2012]


Input: 227x227x3 images
After CONV1: 55x55x96
Second layer (POOL1): $3 \times 3$ filters applied at stride 2

Q: what is the output volume size? Hint: (55-3)/2+1 = 27

## Case Study: AlexNet

[Krizhevsky et al. 2012]


Input: 227x227x3 images
After CONV1: 55x55x96
Second layer (POOL1): $3 \times 3$ filters applied at stride 2
Output volume: 27x27x96
Q: what is the number of parameters in this layer?

## Case Study: AlexNet

[Krizhevsky et al. 2012]


Input: 227x227x3 images
After CONV1: 55x55x96
Second layer (POOL1): $3 \times 3$ filters applied at stride 2
Output volume: 27x27x96
Parameters: 0!

## Case Study: AlexNet

[Krizhevsky et al. 2012]


Input: 227x227x3 images
After CONV1: 55x55x96
After POOL1: 27x27x96

## Case Study: AlexNet

[Krizhevsky et al. 2012]

Full (simplified) AlexNet architecture: [227x227x3] INPUT

[ $55 \times 55 \times 96$ ] CONV1: $9611 \times 11$ filters at stride 4 , pad 0 [27x27x96] MAX POOL1: $3 \times 3$ filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: $2565 \times 5$ filters at stride 1, pad 2
[13x13x256] MAX POOL2: $3 \times 3$ filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: $3843 \times 3$ filters at stride 1, pad 1
[13×13×384] CONV4: $3843 \times 3$ filters at stride 1, pad 1
[13×13x256] CONV5: $2563 \times 3$ filters at stride 1, pad 1
[6x6x256] MAX POOL3: $3 \times 3$ filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

## Case Study: VGGNet [Simonyan and Zisserman, 2014]

## VGGNet

高

Only $3 x 3$ CONV stride 1 , pad 1 and $2 x 2$ MAX POOL stride 2
11.2\% top 5 error in ILSVRC 2013
->
7.3\% top 5 error

## Case Study: VGGNet [Simonyan and Zisserman, 2014]

INPUT: [224x224x3] memory: $224 * 224^{*} 3=150 \mathrm{~K}$ params: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: $224^{*} 224^{*} 64=3.2 \mathrm{M}$ params: $\left(3^{*} 3^{*} 64\right)^{*} 64=36,864$
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: $\left(3^{*} 3^{*} 64\right)^{*} 128=73,728$
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: $\left(3^{*} 3^{*} 128\right)^{*} 128=147,456$
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 $=294,912$
CONV3-256: [56x56x256] memory: $56 * 56 * 256=800 \mathrm{~K}$ params: $\left(3^{*} 3^{*} 256\right)^{*} 256=589,824$
CONV3-256: [56x56x256] memory: 56*56*256=800K params: $\left(3^{*} 3^{*} 256\right)^{*} 256=589,824$
POOL2: [28x28x256] memory: $28^{*} 28^{*} 256=200 \mathrm{~K}$ params: 0
CONV3-512: [28x28x512] memory: $28^{*} 28^{*} 512=400 \mathrm{~K}$ params: $\left(3^{*} 3^{*} 256\right)^{*} 512=1,179,648$
CONV3-512: [28x28x512] memory: $28^{*} 28^{*} 512=400 \mathrm{~K}$ params: $(3 * 3 * 512) * 512=2,359,296$
CONV3-512: [ $28 \times 28 \times 512$ ] memory: $28^{*} 28^{*} 512=400 \mathrm{~K}$ params: $\left(3^{*} 3^{*} 512\right)^{*} 512=2,359,296$
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14×14x512] memory: $14^{*} 14 * 512=100 \mathrm{~K}$ params: $(3 * 3 * 512) * 512=2,359,296$
CONV3-512: [14x14x512] memory: $14 * 14 * 512=100 \mathrm{~K}$ params: $(3 * 3 * 512)^{*} 512=2,359,296$
CONV3-512: [ $14 \times 14 \times 512$ ] memory: $14^{*} 14^{*} 512=100 \mathrm{~K}$ params: $\left(3^{*} 3^{*} 512\right)^{*} 512=2,359,296$
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: $7^{*} 7 * 512 * 4096=102,760,448$
FC: [1x1x4096] memory: 4096 params: $4096 * 4096=16,777,216$
FC: [1x1x1000] memory: 1000 params: $4096 * 1000=4,096,000$

## Case Study: GoogLeNet [Szegedy et al., 2014]



Inception module

ILSVRC 2014 winner (6.7\% top 5 error)

## Case Study: GoogLeNet

| type | patch size/ <br> stride | output <br> size | depth | $\# 1 \times 1$ | $\# 3 \times 3$ <br> reduce | $\# 3 \times 3$ | $\# 5 \times 5$ <br> reduce | $\# 5 \times 5$ | pool <br> proj | params | ops |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| convolution | $7 \times 7 / 2$ | $112 \times 112 \times 64$ | 1 |  |  |  |  |  |  | 2.7 K | 34 M |
| max pool | $3 \times 3 / 2$ | $56 \times 56 \times 64$ | 0 |  |  |  |  |  |  |  |  |
| convolution | $3 \times 3 / 1$ | $56 \times 56 \times 192$ | 2 |  | 64 | 192 |  |  |  | 112 K | 360 M |
| max pool | $3 \times 3 / 2$ | $28 \times 28 \times 192$ | 0 |  |  |  |  |  |  |  |  |
| inception (3a) |  | $28 \times 28 \times 256$ | 2 | 64 | 96 | 128 | 16 | 32 | 32 | 159 K | 128 M |
| inception (3b) |  | $28 \times 28 \times 480$ | 2 | 128 | 128 | 192 | 32 | 96 | 64 | 380 K | 304 M |
| max pool | $3 \times 3 / 2$ | $14 \times 14 \times 480$ | 0 |  |  |  |  |  |  |  |  |
| inception (4a) |  | $14 \times 14 \times 512$ | 2 | 192 | 96 | 208 | 16 | 48 | 64 | 364 K | 73 M |
| inception (4b) |  | $14 \times 14 \times 512$ | 2 | 160 | 112 | 224 | 24 | 64 | 64 | 437 K | 88 M |
| inception (4c) |  | $14 \times 14 \times 512$ | 2 | 128 | 128 | 256 | 24 | 64 | 64 | 463 K | 100 M |
| inception (4d) |  | $14 \times 14 \times 528$ | 2 | 112 | 144 | 288 | 32 | 64 | 64 | 580 K | 119 M |
| inception (4e) |  | $14 \times 14 \times 832$ | 2 | 256 | 160 | 320 | 32 | 128 | 128 | 840 K | 170 M |
| max pool | $3 \times 3 / 2$ | $7 \times 7 \times 832$ | 0 |  |  |  |  |  |  |  |  |
| inception (5a) |  | $7 \times 7 \times 832$ | 2 | 256 | 160 | 320 | 32 | 128 | 128 | 1072 K | 54 M |
| inception (5b) |  | $7 \times 7 \times 1024$ | 2 | 384 | 192 | 384 | 48 | 128 | 128 | 1388 K | 71 M |
| avg pool | $7 \times 7 / 1$ | $1 \times 1 \times 1024$ | 0 |  |  |  |  |  |  |  |  |
| dropout (40\%) |  | $1 \times 1 \times 1024$ | 0 |  |  |  |  |  |  |  |  |
| linear |  | $1 \times 1 \times 1000$ | 1 |  |  |  |  |  |  | 1000 K | 1 M |
| softmax |  | $1 \times 1 \times 1000$ | 0 |  |  |  |  |  |  |  |  |

Fun features:

- Only 5 million params! (Removes FC layers completely)


## Compared to AlexNet:

- 12X less params
- $2 x$ more compute
- 6.67\% (vs. 16.4\%)


## Case Study: ResNet [He et al., 2015]

ILSVRC 2015 winner (3.6\% top 5 error)


## Case Study: ResNet [He et al., 2015]



(slide from Kaiming He)

## Further Reading

Stanford CS231n, lecture 5, Convolutional Neural Networks
http://cs231n.stanford.edu/schedule.html
Deep learning with PyTorch
https://pytorch.org/tutorials/beginner/deep learning 60min blitz.html
AlexNet (2012):
https://papers.nips.cc/paper/2012/hash/c399862d3b9d6b76c8436e924a68c45bAbstract.html
Vgg16 (2014): https://arxiv.org/abs/1409.1556 GoogleNet (2014): https://arxiv.org/abs/1409.4842
ResNet (2015): https://arxiv.org/abs/1512.03385

